

Element 95

Axisymmetric Quadrilateral with Bending

This is the same formulation as element type 10, with bending effects included. Element type 95 provides a capability to do efficient analysis of axisymmetric structures deforming axisymmetrically and in bending. The elements are based on the usual (isoparametric) displacement formulation in the z-r plane, whereas in the circumferential direction sinusoidal variation is assumed, which can be expressed by:

$$u_z(\theta) = u_z(1 + \cos \theta)/2 + \bar{u}_z(1 - \cos \theta)/2$$

$$u_r(\theta) = u_r(1 + \cos \theta)/2 + \bar{u}_r(1 - \cos \theta)/2$$

$$u_\theta(\theta) = u_\theta \sin \theta$$

The element is integrated numerically in the z-r plane using the usual Gaussian quadrature formulas, whereas numerical integration with an equidistant scheme is used along the circumference. The number of points along the circumference is chosen with the [SHELL SECT](#) parameter, and must be at least equal to 3. For linear elastic material behavior, this element furnishes “exact” results (for the circumferential variation) for axisymmetric and bending deformation even with the minimum number of circumferential integration points.

Because of the numerical integration scheme, the elements can also be used if material nonlinearity (creep or plasticity) plays a role. It should be noted that the exact solution does not necessarily contain the sinusoidal variation as given above, and, in that sense, the solution obtained is an approximate one. However, experience obtained so far indicates that for thick-walled members, where ovalization of the cross section does not play a significant role, the solution is sufficiently accurate for most practical purposes. Note that if nonlinear effects are present, the number of integration points along the circumference should be at least 5, and more if desired.

This element cannot be used with the [CONTACT](#) option; use gap element type 97 instead.

Quick Reference

Type 95

Axisymmetric, arbitrary ring with a quadrilateral cross section and bending effects included. This is achieved by including additional degrees of freedom representing the displacements at the point 180° along the circumference.

Connectivity

Four nodes per element. Node numbering follows right-handed convention (counterclockwise). See [Figure 3-150](#).

Geometry

No geometry input is required for this element.

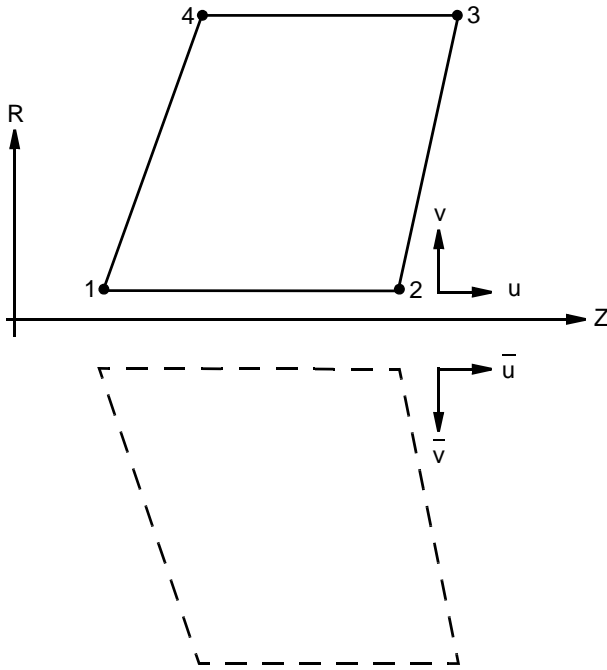


Figure 3-150 Axisymmetric Ring with Bending

Coordinates

Two coordinates in the global z and r directions.

Degrees of Freedom

Global displacement degrees of freedom:

- 1 = $u = z$ displacement (along symmetry axis).
- 2 = $v =$ radial displacement.
- 3 = $\bar{u} = z$ displacement of reverse side.
- 4 = $\bar{v} =$ radial displacement of reverse side.
- 5 = $w =$ circumferential displacement at 90° angle.

Distributed Loads

Load types for distributed loads are as follows:

Load Type	Description
0,50	Uniform pressure distributed on 1-2 face of the element.
1,51	Uniform body force per unit volume in first coordinate direction.
2,52	Uniform body force by unit volume in second coordinate direction.

Load Type	Description
3,53	Nonuniform pressure on 1-2 face of the element.
4,54	Nonuniform body force per unit volume in first coordinate direction.
5,55	Nonuniform body force per unit volume in second coordinate direction.
6,56	Uniform pressure on 2-3 face of the element.
7,57	Nonuniform pressure on 2-3 face of the element.
8,58	Uniform pressure on 3-4 face of the element.
9,59	Nonuniform pressure on 3-4 face of the element.
10,60	Uniform pressure on 4-1 face of the element.
11,61	Nonuniform pressure on 4-1 face of the element.
20	Uniform shear force on 1-2 face in the 1-2 direction.
21	Nonuniform shear force on side 1-2.
22	Uniform shear force on side 2-3 (positive from 2 to 3).
23	Nonuniform shear force on side 2-3.
24	Uniform shear force on side 3-4 (positive from 3 to 4).
25	Nonuniform shear force on side 3-4.
26	Uniform shear force on side 4-1 (positive from 4 to 1).
37	Nonuniform shear force on side 4-1.
100	Centrifugal load, magnitude represents square of angular velocity [rad/time]. Rotation axis specified in the ROTATION A option.
102	Gravity loading in global direction. Enter three magnitudes of gravity acceleration in respectively global, x-, y-, z-direction.
103	Coriolis and centrifugal load; magnitude represents square of angular velocity [rad/time]. Rotation axis is specified in the ROTATION A option.

For all nonuniform loads, the load magnitude is supplied via the [FORCEM](#) user subroutine.

All pressures are positive when directed into the element. In addition, point loads can be applied at the nodes. The load types 0-11 correspond to axisymmetric loading. The load type 50-61 correspond to “bending” loads. In that case, the circumferential variation of the distributed load is equal to $p(\theta) = p_0 \cos(\theta)$.

For other types of distributed loads that are normally applicable for all types of elements, please refer to [Distributed Loads](#) in Chapter 1 of this manual.

Output of Strains

Output of strains at the centroid of the element or at the Gauss points in global coordinates is:

$$1 = \varepsilon_{zz}$$

$$2 = \varepsilon_{rr}$$

$$3 = \varepsilon_{\theta\theta}$$

$$4 = \gamma_{rz}$$

$$5 = \gamma_{z\theta}$$

$$6 = \gamma_{\theta r}$$

Output of Stresses

Same as for [Output of Strains](#).

Transformation

The transformation on degrees of freedom 3 and 4 are the same as on degrees of freedom 1 and 2. Four global degrees of freedom can be transformed into local coordinates.

Tying

Use the [UFORMSN](#) user subroutine.

Output Points

Output is available at the centroid or at the 4 Gaussian points shown in [Figure 3-151](#).

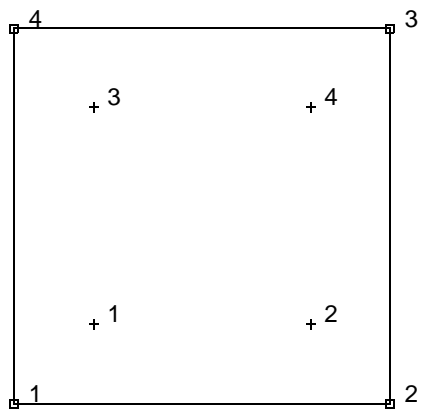


Figure 3-151 Integration Point Locations

Integration Along Circumference

The element is integrated numerically in the circumferential direction. The number of integration points is given on the **SHELL SECT** parameter. The points are equidistant.

Large Displacement

This element has only geometrically linear behavior. Neither the **LARGE DISP** or the **LARGE STRAIN** parameter has any effect on this element.