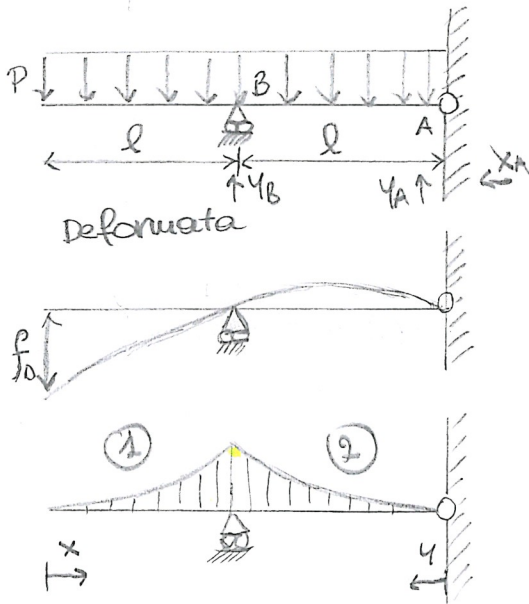


Reazioni Vincolari



(1) $y_A + y_B = 2pl$

(2) $x_A = 0$

(A) $2pl \cdot l - y_B \cdot l = 0$

$$\begin{cases} x_A = 0 \\ y_B = 2pl \\ y_A = 0 \end{cases}$$

TRATTO ① $[0 \leq x \leq l]$

$M_{F_1}(x) = \frac{Px^2}{2}$

TRATTO ② $[0 \leq y \leq l]$

$M_{F_2}(y) = \frac{Py^2}{2}$

Momento flettente massimo

$$M_{F,B} = M_{F,max} = \frac{Pl^2}{2}$$

Applica il th. di Castigliano per il calcolo della freccia f_D
 • SOVRAPPOSIZIONE DEGLI EFFETTI

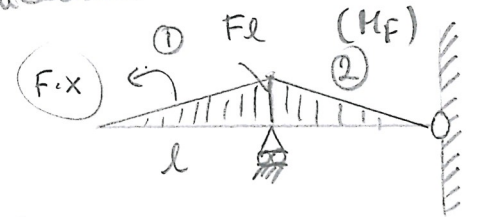
→ Reazioni Vincolari

(1) $y_B + y_A = F$

(A) $2Fl = y_B \cdot l \Rightarrow y_B = 2F$

$\Rightarrow y_A = -F$

Si mantiene la simmetria del concavetto



→ M_F TOTALE: $M_{F_1}(x) = \frac{Px^2}{2} + Fx$
 \Downarrow
 $\equiv M_{F_2}(y)$

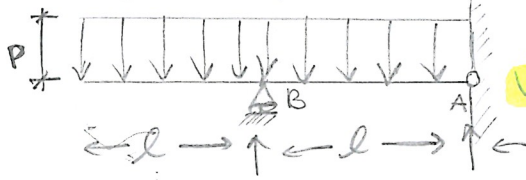
Voluto l'energia: $U = U_1 + U_2 \Rightarrow U_1 = U_2$

$\Rightarrow U = 2U_1 = 2 \cdot \frac{1}{2EI} \int_0^l \left(\frac{Px^2}{2} + Fx \right)^2 dx = \frac{1}{EI} \int_0^l \left[\frac{P^2x^4}{4} + F^2x^2 + PFx^3 \right] dx$

$\Rightarrow \frac{1}{EI} \left[\frac{P^2x^5}{20} + \frac{F^2x^3}{3} + \frac{PFx^4}{4} \right]_0^l \Rightarrow \frac{1}{EI} \left(\frac{P^2l^5}{20} + \frac{F^2l^3}{3} + \frac{PF^2l^4}{4} \right) = U$

$\Rightarrow f_D = \frac{\partial U}{\partial F} \Big|_{F=0} = \frac{1}{EI} \cdot \frac{Pl^4}{4} \Rightarrow f_D = \frac{Pl^4}{4EI}$

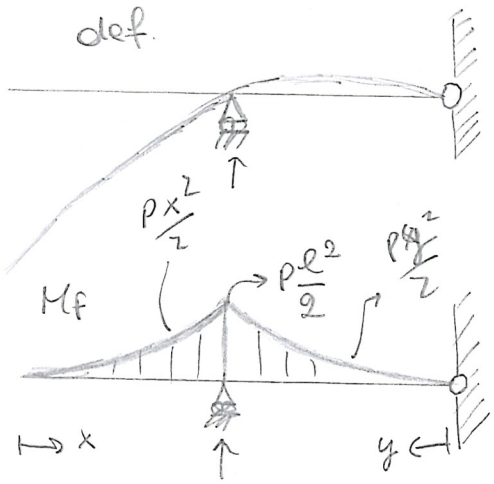
ES 3 → 11/06/2014 - TRACCE C/D ⇒ VALORI LA ROTAZIONE IN D



REAZIONI VINCOLARI ⇒

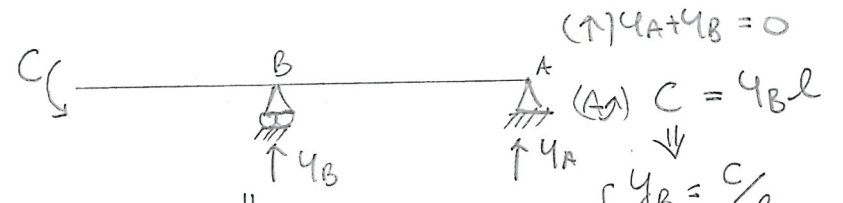
$$\begin{aligned} (1) & y_A + y_B = 2pl \\ (\rightarrow) & x_A = 0 \\ (A) & 2pl^2 = 4Bl \end{aligned}$$

$$\Rightarrow \begin{cases} x_A = 0 \\ y_B = 2pl \\ y_A = 0 \end{cases}$$



Considera la coppia C x il calcolo della rotazione in D. (φ_D)

→ Sfrutto la sovrapposizione degli effetti



$M_f \rightarrow$ TOTALE

TRATTO ① $\rightarrow x$

$$M_{f1}(x) = \frac{px^2}{2} + C$$

TRATTO ② $\leftarrow y$

$$M_{f2}(y) = \frac{py^2}{2} + \frac{C}{2}y$$

$$U_1 = \frac{1}{2EI} \int_0^l \left(\frac{p^2 x^4}{4} + C^2 + px^2 C \right) dx = \frac{1}{2EI} \left[\frac{p^2 l^5}{20} + C^2 l + \frac{p C l^3}{3} \right]$$

$$U_2 = \frac{1}{2EI} \int_0^l \left(\frac{p^2 y^4}{4} + \frac{C^2}{2} y^2 + \frac{p C}{2} y^3 \right) dy = \frac{1}{2EI} \left[\frac{p^2 l^5}{20} + \frac{C^2}{2} \frac{l^3}{3} + \frac{p C}{2} \frac{l^4}{4} \right]$$

$$U = U_1 + U_2 = \frac{1}{2EI} \left[\frac{p^2 l^5}{10} + \frac{4}{3} C^2 l + \left(\frac{4+3}{12} \right) p C l^3 \right]$$

$$= \frac{1}{2EI} \left[\frac{p^2 l^5}{10} + \frac{4}{3} C^2 l + \frac{7}{12} p C l^3 \right]$$

$$\varphi_D = \frac{\partial U}{\partial C} \Big|_{C=0} = \frac{1}{2EI} \left[\frac{7}{12} p l^3 \right] \Rightarrow \varphi_D = \frac{7}{24EI} p l^3$$

ROTAZIONE ANTIORARIA

Momento flettente massimo come da tracce A/B

oppure da * con $C=0$, $x=l$ o $y=l$

$$M_{f \max} = \frac{pl^2}{2}$$