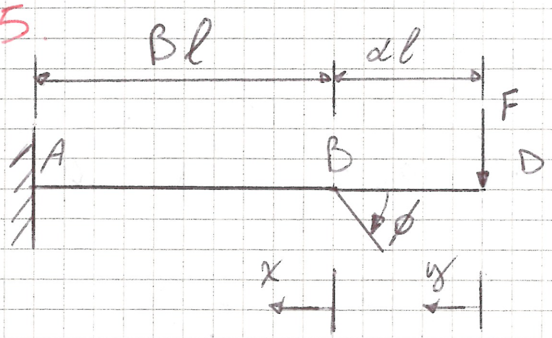
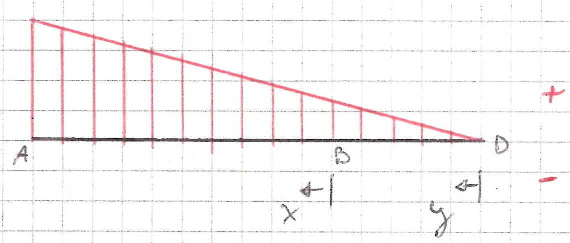
Esercizio 2.15



Determinare ϕ con il teorema di Castigliano.

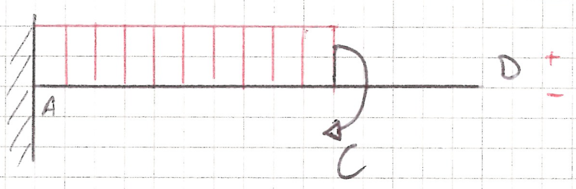
Ricavo subito $Mf_F(x)$ e $Mf_F(y)$:



$$Mf_F(x) = F\cdot(x+\alpha l)$$

$$Mf_F(y) = F\cdot y$$

Ora devo introdurre una coppia C fittizia nel punto B.



Calcolo Mf_C

$$Mf_C(x) = C$$

$$Mf_C(y) = 0$$

Imposto ora la risoluzione:

$$U = \frac{1}{2EJ} \cdot \left[\int_0^{\beta l} \left(F(x + \alpha l) + C \right)^2 dx + \int_0^{\alpha l} (F \cdot y)^2 dy \right] =$$

$$= \frac{1}{2EJ} \left[\int_0^{\beta l} \left(F^2 x^2 + F^2 \alpha^2 l^2 + C^2 + 2FCx + 2F\alpha l C + 2F^2 x \cdot \alpha l \right) dx + \int_0^{\alpha l} (F^2 y^2)\, dy = \right.$$

$$= \frac{1}{2EJ} \cdot \left[\frac{F^2 \beta^3 l^3}{3} + F^2 \alpha^2 \beta l^3 + C^2 \beta l + \cancel{2} FC \cdot \frac{\beta^2 l^2}{\cancel{2}} + 2FC \cdot \alpha \beta l^2 + \frac{2F^2\alpha \beta^2 l^3}{2} + \right.$$

$$\left. + F^2 \frac{\alpha^3 l^3}{3} \right]$$

$$\phi = \left. \frac{\partial U}{\partial C} \right|_{C=0} = \frac{1}{2EJ} \cdot \left(F \cdot \beta^2 l^2 + 2F \cdot \alpha \beta l^2 \right) = \frac{F l^2}{EJ} \cdot \left(\frac{\beta^2}{2} + \alpha \beta \right)$$