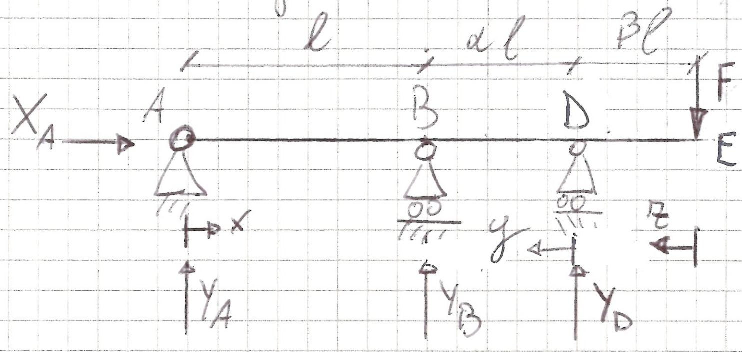
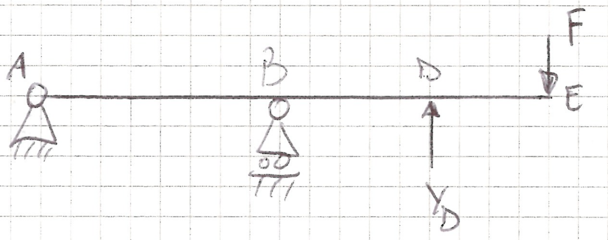
Esercizio 2.01

Risolvere la seguente struttura staticamente indeterminata.

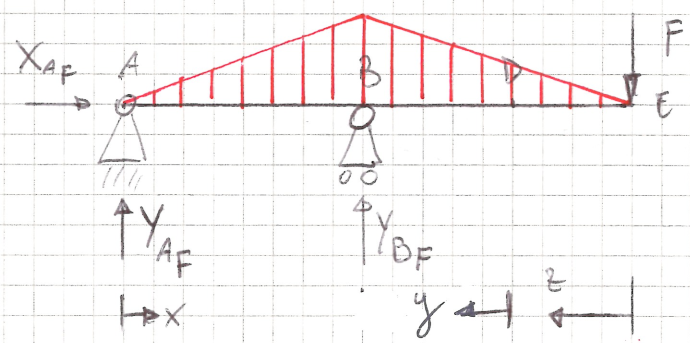


Si consideri Y_D come variabile iperstatica:



Risolvo applicando il PLV.

Considero solo F:



$\overset{+}{\rightarrow}]\ X_{AF} = 0$

$\overset{+}{\uparrow}]\ Y_{AF} + Y_{BF} = F \longrightarrow Y_{BF} = F + F(\alpha+\beta) = F(\alpha+\beta+1)$

$\overset{+}{\circlearrowleft} B)]\ Y_{AF} = -F(\alpha+\beta)$

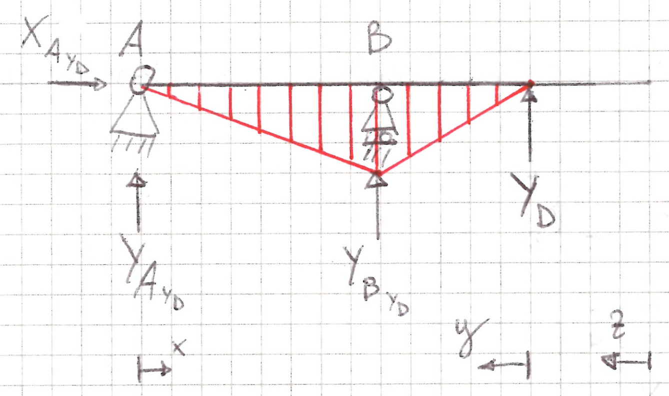
Calcolo i vari Mf_F, positivo se pone a trazione le fibre superiori.

$$\underset{\overline{AB}}{Mf_F(x)} = F(\alpha+\beta)\cdot x$$

$$\underset{\overline{BD}}{Mf_F(y)} = F(\beta l + y)$$

$$\underset{\overline{ED}}{Mf_F(z)} = F\cdot z$$

Considero la sola reazione iperstatica Y_D:



$\overset{+}{\rightarrow}]$ $X_{A_{Y_D}} = 0$

$\uparrow +]$ $Y_{A_{Y_D}} + Y_{B_{Y_D}} + Y_D = 0 \rightarrow Y_{B_{Y_D}} = -Y_D(1+\alpha)$

$\overset{+}{\circlearrowleft}_B]$ $Y_{A_{Y_D}} = Y_D \cdot \alpha$

Mf negativo, in quanto tende le fibre sotto.

$Mf_{Y_D}(x) = -Y_D \cdot \alpha \cdot x$

$Mf_{Y_D}(y) = -Y_D \cdot y$

$Mf_{Y_D}(z) = 0$

Inserisco il carico esplorativo al posto di Y_D e ottengo.

$Mf_1(x) = -1 \cdot \alpha \cdot x$

$Mf_1(y) = -1 \cdot y$

$Mf_1(z) = 0$

Applico il PLV

$$0 = \cancel{\frac{1}{EJ}}\left[\int_0^l \left(F(\alpha+\beta)x - Y_D \cdot \alpha \cdot x\right)(-\alpha x)\,dx + \int_0^{\alpha l}\left(F(\beta l + y) - Y_D \cdot y\right)(-y)\,dy + \cancel{\int_0^{\beta l}(F \cdot z + 0) \cdot (0)\,dz}\right]$$

$$0 = \int_0^l \left(-\alpha x^2 F(\alpha+\beta) + \alpha^2 Y_D x^2\right)dx + \int_0^{\alpha l}\left(-F\beta l \cdot y - F \cdot y^2 + Y_D y^2\right)dy$$

$$0 = -\alpha F(\alpha+\beta)\frac{l^3}{3} + \alpha^2 Y_D \frac{l^3}{3} - F\beta l \frac{\alpha^2 l^2}{2} - F\frac{\alpha^3 l^3}{3} + Y_D \frac{\alpha^3 l^3}{3}$$

$$0 = -\alpha F(\alpha+\beta)\cdot\frac{1}{3} + \alpha^2 Y_D \cdot \frac{1}{3} - F\cdot\beta\cdot\frac{\alpha^2}{2} - F\cdot\frac{\alpha^3}{3} + Y_D\,\frac{\alpha^3}{3}$$

$$\frac{(\alpha^2+\beta\alpha)}{3}F + F\,\frac{\beta\alpha^2}{2} + F\,\frac{\alpha^3}{3} = \frac{\alpha^2 Y_D}{3} + Y_D\,\frac{\alpha^3}{3}$$

$$F\left(\frac{\alpha^2+\beta\alpha}{3} + \frac{\beta\alpha^2}{2} + \frac{\alpha^3}{3}\right) = Y_D\left(\frac{\alpha^2}{3} + \frac{\alpha^3}{3}\right)$$

Ricavo quindi Y_D inserendo "le cifre" i,j,k in α e β.

$$X_A = X_{AY_D} + X_{AF} = 0$$

$$Y_A = Y_{AY_D} + Y_{AF} = Y_D\cdot\alpha - F(\alpha+\beta)$$

$$Y_B = -Y_D(1+\alpha) + F(\alpha+\beta+1)$$